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Exact renormalization group equation in presence of rescaling anomaly

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Abstract: Wilson's approach to renormalization group is reanalyzed for supersymmetric Yang-Mills theory. The usual demonstration of exact renormalization group equation must be modified due to the presence of the so called Konishi anomaly under the rescaling of superfields. We carry out the explicit computation for $N = 1$ SUSY Yang-Mills theory with the simpler, gauge invariant regularization method, recently proposed by Arkani-Hamed and Murayama. The result is that the Wilsonian action S_M consists of two terms, i.e. the non anomalous term, which obeys Polchinski' s flow equation and Fujikawa-Konishi determinant contribution. This latter is responsible for Shifman-Vainshtein relation giving the exact β-function.

Keywords: Renormalization Regularization and Renormalons, Nonperturbative Effects, Extended Supersymmetry.

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Contents

1. Introduction

In two interesting papers [1, 2] Arkani-Hamed and Murayama have analyzed the problem of renormalization group (RG) invariance of the so called exact results in SUSY gauge field theory. They have shown convincingly the mechanism by which Shifman-Vainshtein's results on β-function for $N = 1$ SUSY Yang-Mills (SYM) theory come out. In [2] they have proposed an explicit regularization method for such a theory which is free of the usual problem of the inconsistency with local gauge symmetry [5, 6]. Although this method has its own problems seeing that it depends on $N = 4$ SYM, it permits the great simplification in theoretical investigation such as the derivation of exact renormalization group equation (ERGE) (assuming that the regularization through "finite" $N = 4$ theory is basically sound).

In this note, we reformulate the results of [1, 2] in the spirit of ERGE due to Polchinski [3], which is the formal way to apply Wilson's decimation method to the continuum field theory [4].

We note, first of all, that the vacuum energy of auxiliary chiral superfields (belonging to $N = 4$ theory) in the presence of external vector fields, being in principle dependent on the relevant gauge fields, cannot be discarded as in [3]. We apply the known result on rescaling anomaly [8] for chiral superfields to evaluate explicitly such a vacuum contribution.

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This leads to the conclusion that the Wilsonian action S_M, M being a cutoff mass, consists of two parts, the normal term which obeys the RG flow equation of the type proposed in [3] and Fujikawa-Konishi determinant contribution [7, 8]; it is the last term which is responsible for the Shifman-Vainshtein result on the β-function. These results are exact and they neither depend on the perturbation expansion (except for the calculation of anomaly which is, however, immune to the radiative corrections), nor on the large M approximation.

2. Regularization of $N = 1$ SYM theory and ERG equation

2.1 Regularized $N = 1$ SYM

We have to regularize the $N = 1$ SYM theory whose classical action is

$$S(V) = \frac{1}{16}\int d^4x\, d^2\theta \frac{1}{g_h^2} W_a^\alpha W_\alpha^a + \text{h.c.} \tag{2.1}$$

This is the so called holomorphic representation, which is written in a manifestly holomorphic way in the gauge coupling

$$\frac{1}{g_h^2} = \frac{1}{g^2} + \frac{i\theta}{8\pi^2} \tag{2.2}$$

(throughout the present note, we adopt the conventions used in [1, 2, 9]).

The more conventional "canonical representation"

$$S_c(V_c) = \frac{1}{16}\int d^4x\, d^2\theta \left(\frac{1}{g_c^2} + \frac{i\theta}{8\pi^2}\right) W_a^\alpha(g_c V_c) W_\alpha^a(g_c V_c) + \text{h.c.} \tag{2.3}$$

can be obtained from eq. (2.1) by the rescaling transformation. For details, see [2].

To quantize eq. (2.1) with suitable cut-off, we follow [2] and start from the $N = 4$ SYM in $4D$ which is believed to be finite [15]. Apart from the ghosts necessary for gauge invariance, this latter consists of vector superfield V and chiral and anti-chiral triplets $(\varphi_i, \bar\varphi_i)_{i=1,2,3}$ $(\bar D_{\dot\alpha}\varphi_i = 0)$.

All the superfields are in the adjoint representation of the gauge group G.

The classical action is

$$\begin{aligned} S_{N=4}(V,\varphi,\bar\varphi; g) = \frac{1}{16}\int d^4x\, d^2\theta \frac{1}{g^2} W_\alpha^a W_a^\alpha + \text{h.c.} + \int dx\, d^4\theta\, \mathcal{R}e\left(\frac{2}{g^2}\right) t_2(A) \bar\varphi^i e^V \varphi^i + \\ + \int d^4x\, d^2\theta\, \mathcal{R}e\left(\frac{1}{g^2}\right) \sqrt{2}\, Tr\left(\varphi^i\left[\varphi^j, \varphi^k\right]\right) \frac{\epsilon^{ijk}}{3!} + \text{h.c.} \end{aligned} \tag{2.4}$$

where $t_2(A)$ is the Dynkin index of the adjoint representation of G.

Following [2] we write down the regulated $N = 1$ SYM theory as

$$Z_0(J_i, \bar{J}_i, J_V) = \int \mathcal{D}[V] \int \prod_{i=1}^{3} \mathcal{D}[\varphi_i]\, \mathcal{D}[\bar{\varphi}_i] \exp i \Big[S_{N=4}(V, \varphi, \bar{\varphi}; g_0) + \frac{M_0}{2} \int \varphi_i^2 + \\ + \text{h.c.} + \int J_i \varphi_i + \text{h.c.} + \int J_V V \Big], \qquad (2.5)$$

where $\int \varphi_i^2$ stands for $\int d^4p\, d^2\theta\, \sum_{a,i} \varphi_i^a(-p) \varphi_i^a(p)$.

This expression must be really supplemented with appropriate gauge fixing term and corresponding "ghosts". But as far as our regularization is concerned, these terms and auxiliary fields do not influence the arguments in any essential way. This is not the case in more conventional cut-off method [3, 5, 6].

The mass term

$$\frac{M_0}{2} \int d^4x\, d^2\theta\, \varphi_i^2 + \text{h.c.} \qquad (2.6)$$

breaks $N = 4$ SUSY but preserves the finiteness of the RHS of eq. (2.5) (soft breaking [16]). This term is gauge invariant.[1]

From a physical point of view, one takes large enough M_0 with respect to the momentum scale $p \sim M'$ which characterizes low energy physics. Thus, at $p \sim M' \ll M_0$, it is expected that the influences of auxiliary superfields φ_i and $\bar{\varphi}_i$ are negligible.

2.2 ERG equation

The first step for applying Wilson's idea of RG is to vary M_0 to $M < M_0$ and search for the new action functional (Wilsonian action) $S_M(V, \varphi_i, \bar{\varphi}_i)$ in such a way that the physics is unchanged lowering the regularizing mass, i.e.

$$Z_M(J_i, \bar{J}_i, J_V) = \int \mathcal{D}[V] \int \prod_{i=1}^{3} \mathcal{D}[\varphi_i]\, \mathcal{D}[\bar{\varphi}_i] \exp i \Big[S_M(V, \varphi, \bar{\varphi}) + \frac{M}{2} \int \varphi_i^2 + \\ + \text{h.c.} + \int f(M) J_i\, \varphi_i + \text{h.c.} + \int J_V V \Big] \qquad (2.7)$$

(where f(M) is a renormalization factor to be determined) must be equal to the original Z_0 (eq. (2.5)).

We derive the equation which should be obeyed by S_M following Polchinski [3]. Let us write down the "trivial" equality

$$0 = \int d^4p\, M \partial_M C(p, M) \int \mathcal{D}[V] \int \prod_{i=1}^{3} \mathcal{D}[\varphi_i]\, \mathcal{D}[\bar{\varphi}_i] \Big\{ \int d^2\theta \frac{\delta}{\delta \varphi_i(p)} \Big[-iM \varphi_i(p) + \\ + \frac{1}{2} \frac{\delta}{\delta \varphi_i(-p)} \Big] + \text{h.c.} \Big\} \exp\left(i S_{tot}\right), \qquad (2.8)$$

with $S_{tot} \doteq S_M + \frac{M}{2} \int \varphi_i^2 + \text{h.c.} \int f(M) J_i \varphi_i + \text{h.c.} + \int J_V V$.

[1] This expression differs from the one in [2] by the Dynkin index of the adjoint representation of the gauge group.

The cut-off function $C(p,M)$ is simply equal to M^{-1} in our case.

To go further, however, we must decide what to do with the singular term

$$\delta_{ii} \doteq \sum_{i,a} \frac{\delta\varphi_i^a(p)}{\delta\varphi_i^a(p)} \,. \tag{2.9}$$

In [3], a similar term is discarded because it adds only a constant (perhaps divergent) overall factor to Z_M.

In our case, apart from being singular, it may depend on the V superfield.

In other words, one cannot ignore the vacuum energy of φ_i and $\bar{\varphi}_i$ superfields in external V superfield since it depends, in general, on V.

Thus it is necessary to analyze the quantity δ_{ii} and find its exact value within our regularization scheme.

To this end, we apply the result on rescaling anomaly studied by Konishi and Schizuya [8]. Under the (infinitesimal) rescaling transformation of chiral superfields φ_i in the presence of external gauge superfield V

$$\varphi_i = e^{\delta\alpha}\varphi_i' \tag{2.10}$$

(here one must consider $\delta\alpha(x)$ as a chiral superfield) one has a non trivial Jacobian

$$\det\left(\frac{\delta\varphi}{\delta\varphi'}\right) = 1 + i\int d^4x\, d^2\theta\, \delta\alpha(x)\frac{1}{8}\frac{t_2(A)}{8\pi^2}W_a^{\alpha}W_{\alpha}^{a} \,. \tag{2.11}$$

where $t_2(A)$ is the Dynkin index of the adjoint representation of G.

From eq. (2.11) one readily obtains the anomalous Ward-Takahashi identity or "Konishi equation"

$$\left\langle \int \varphi_i^a(p)\frac{\delta S_{tot}}{\delta\varphi_i^a(p)} + \frac{1}{8}\frac{t_2(A)}{8\pi^2}\int W_a^{\alpha}W_{\alpha}^{a}\right\rangle = 0 \,. \tag{2.12}$$

Now one can relate the anomalous term above to the singular quantity "δ_{ii}" (or its integral).

Since

$$\begin{aligned} 0 &= \int d^4p\, d^2\theta \frac{1}{i}\frac{\delta}{\delta J_i(p)}\int \mathcal{D}[V]\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i]\,\mathcal{D}[\bar{\varphi}_i]\frac{\delta}{\delta\varphi_i(p)}\exp\left(iS_{tot}\right) \\ &= \int d^4p\, d^2\theta\int \mathcal{D}[V]\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i]\,\mathcal{D}[\bar{\varphi}_i]\frac{\delta}{\delta\varphi_i(p)}\left(\varphi_i(p)\exp\left(iS_{tot}\right)\right) \end{aligned} \tag{2.13}$$

one immediately obtains

$$\int\sum_i \delta_{ii} = \left\langle \frac{i}{8}\int d^2\theta\, \frac{3t_2(A)}{8\pi^2}W_{\alpha}^{a}W_{a}^{\alpha}\right\rangle . \tag{2.14}$$

With this result, one can proceed with the derivation of ERG equation.

Eq. (2.8) becomes

$$0 = \int d^4p \left(-\frac{1}{M}\right) \int \mathcal{D}[V] \int \prod_{i=1}^{3} \mathcal{D}[\varphi_i]\, \mathcal{D}[\bar{\varphi}_i] \Big\{ \int d^2\theta \Big[- iM\delta_{ii} + M\varphi_i(p) \times \tag{2.15}$$
$$\times \frac{\delta S_{tot}}{\delta\varphi_i(p)} + \frac{1}{2}\left(i\frac{\delta^2 S_{tot}}{\delta\varphi_i(p)\delta\varphi_i(-p)} - \frac{\delta S_{tot}}{\delta\varphi_i(p)}\frac{\delta S_{tot}}{\delta\varphi_i(-p)} \right) \Big] + \text{h.c.} \Big\} \exp\left(iS_{tot}\right).$$

Separating S_M term explicitly from S_{tot}, eq. (2.15) becomes, after a little algebra,

$$0 = \int d^4p \int \mathcal{D}[V] \int \prod_{i=1}^{3} \mathcal{D}[\varphi_i]\, \mathcal{D}[\bar{\varphi}_i] \Big\{ \int d^2\theta \Big[\frac{i}{2}\delta_{ii} -$$
$$-M\partial_M \left(\frac{1}{2}M\varphi_i^2\right) - \frac{1}{2M}\left(i\frac{\delta^2 S_M}{\delta\varphi_i(p)\delta\varphi_i(-p)} - \frac{\delta S_M}{\delta\varphi_i(p)}\frac{\delta S_M}{\delta\varphi_i(-p)} \right) -$$
$$-\frac{f^2(M)}{2M} J_i(-p)J_i(p) - f(M)J_i(-p)\,\varphi_i(p) \Big] + \text{h.c.} \Big\} \exp\left(iS_{tot}\right). \tag{2.16}$$

Note that the term proportional to $i\delta_{ii} = -\frac{1}{8}\int \frac{3t_2(A)}{8\pi^2}W^2$ is contributed also by the double derivative term $\frac{\delta^2}{\delta\varphi_i(p)\delta\varphi_i(-p)}\left(\frac{1}{2}M\int\varphi_i^2\right)$, part of $\frac{\delta^2 S_{tot}}{\delta\varphi_i(p)\delta\varphi_i(-p)}$ term in eq. (2.15), which explains the factor 1/2 in front of δ_{ii} in eq. (2.16).
From the inspection of eq. (2.16) one obtains the following conditions for the M invariance of low energy physics:

1. S_M satisfies:

$$\frac{1}{2M}\int dp\, d^2\theta \left(-\frac{\delta S_M}{\delta\varphi_a^i(p)}\frac{\delta S_M}{\delta\varphi_a^i(-p)} + i\frac{\delta^2 S_M}{\delta\varphi_a^i(p)\delta\varphi_a^i(-p)} \right) + \tag{2.17}$$
$$+\frac{3}{16}\int dp\, d^2\theta\, \frac{t_2(A)}{8\pi^2} W_a^\alpha W_\alpha^a + \text{h.c.} = M\partial_M S_M .$$

2. Also:

$$M\partial_M f(M) = f(M) \qquad \text{i.e.} \qquad f(M) = \frac{M}{M_0}. \tag{2.18}$$

If the conditions eq. (2.17) and eq. (2.18) are satisfied then eq. (2.16) is seen to be equivalent to

$$\int \mathcal{D}[V] \int \prod_{i=1}^{3} \mathcal{D}[\varphi_i]\, \mathcal{D}[\bar{\varphi}_i] \left[-M\partial_M S_{tot} - \frac{f^2(M)}{2M}\int J_i(-p)J_i(p) + \text{h.c.} \right] \exp\left(iS_{tot}\right) = 0\,, \tag{2.19}$$

i.e.

$$M\partial_M Z_M + \left(i\frac{f^2(M)}{2M}\int J_i(-p)J_i(p) + \text{h.c.} \right) Z_M = 0\,, \tag{2.20}$$

i.e.

$$M\partial_M \left[\exp i\left(\frac{M}{2M_0^2}\int J_i(-p)J_i(p) + \text{h.c.} \right) Z_M \right] \doteq M\partial_M \tilde{Z}_M = 0\,. \tag{2.21}$$

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The last equation implies that Z_M is unchanged for varying value of M, except for the tree level two point function, i.e.

$$\tilde{Z}_0 \doteq \exp i\left(\frac{1}{2M_0}\int J_i^2 + \text{h.c.}\right) Z_0 = \exp i\left(\frac{M}{2M_0^2}\int J_i^2 + \text{h.c.}\right) Z_M = \tilde{Z}_M . \tag{2.22}$$

The flow equation (2.17) can be written as

$$M\partial_M\left\{S_M - \frac{1}{16}\int d^2\theta\, \frac{3t_2(A)}{8\pi^2} W_a^\alpha W_\alpha^a \log\left(\frac{M}{M_0}\right) - \text{h.c.}\right\} = \tag{2.23}$$

$$= \frac{1}{2M}\int dp\, d^2\theta \left(i\frac{\delta^2 S_M}{\delta\varphi_a^i(p)\delta\varphi_a^i(-p)} - \frac{\delta S_M}{\delta\varphi_a^i(p)}\frac{\delta S_M}{\delta\varphi_a^i(-p)}\right) + \text{h.c.} \tag{2.24}$$

Since the anomalous term in eq. (2.23) does not depend on φ_i or $\bar{\varphi}_i$, one can express this result as

$$M\partial_M \tilde{S}_M = \frac{1}{2M}\int dp\, d^2\theta \left(i\frac{\delta^2 \tilde{S}_M}{\delta\varphi_a^i(p)\delta\varphi_a^i(-p)} - \frac{\delta \tilde{S}_M}{\delta\varphi_a^i(p)}\frac{\delta \tilde{S}_M}{\delta\varphi_a^i(-p)}\right) + \text{h.c.}$$

$$\tilde{S}_M \doteq S_M - \frac{1}{16}\int d^2\theta\, \frac{3t_2(A)}{8\pi^2} W_a^\alpha W_\alpha^a \log\left(\frac{M}{M_0}\right) - \text{h.c.}\,, \tag{2.25}$$

with the initial condition

$$S_{M_0} = \tilde{S}_{M_0} = S_{N=4}(V, \varphi_i, \bar{\varphi}_i; g_0)\,. \tag{2.26}$$

2.3 Change in holomorphic coupling constant

The low energy physics of $N = 1$ SYM (at $p \sim M' \ll M < M_0$) is given by $S_M(V, 0, 0)$.

The normal part $\tilde{S}_M(V, 0, 0)$ (see eq. (2.25)) is expected to vary rather slowly, i.e.

$$\tilde{S}_M(V, 0, 0) \simeq \tilde{S}_{M_0}(V, 0, 0) + \mathcal{O}\left(\frac{1}{M}\right). \tag{2.27}$$

Then

$$S_M(V, 0, 0) \simeq \frac{1}{16}\int d^4x\, d^2\theta \left(\frac{1}{g_0^2} + \frac{3t_2(A)}{8\pi^2}\log\left(\frac{M}{M_0}\right)\right) W_a^\alpha W_\alpha^a + \text{h.c.} \tag{2.28}$$

i.e., for $N = 1$ SYM theory, the holomorphic gauge coupling constant changes according to

$$\frac{1}{g^2(M_2)} - \frac{1}{g^2(M_1)} = \frac{3t_2(A)}{8\pi^2}\log\left(\frac{M_2}{M_1}\right) \tag{2.29}$$

or, in term of β function

$$M\partial_M\left(\frac{1}{g^2(M)}\right) = \frac{3t_2(A)}{8\pi^2}\,. \tag{2.30}$$

We refer to [1, 2] for the detail of how to go over to the "canonical form" from eq. (2.30) and to obtain the "exact" expression of the β function due to Novikov, Shifman, Vainshtein and Zakarov [10, 11].

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3. The representation for S_M

3.1 The solution of ERG equation

To have more accurate estimate of $\tilde{S}_M$ from eq. (2.25), it would be convenient to have a representation of S_M (or $\tilde{S}_M$) which makes its physical interpretation more explicit.

The formal solution of eq. (2.25) with the given initial condition is

$$\exp\left(i\tilde{S}_M(V,\varphi_i,\bar{\varphi}_i;g)\right) = \exp\left\{\frac{i}{2}\int\left(\frac{1}{M_0}-\frac{1}{M}\right)\frac{\delta^2}{\delta\varphi^i_a(p)\delta\varphi^i_a(-p)}+\text{h.c.}\right\}\times \\ \times\exp\left(iS_{N=4}(V,\varphi_i,\bar{\varphi}_i;g_0)\right). \tag{3.1}$$

If one applies the equality for the gaussian integral

$$\exp\left(\frac{1}{2}\lambda^{-1}_{ij}\partial_{a_i}\partial_{a_j}\right)f(a) = \frac{\int_{\mathbf{R}^N}\prod_{i=1}^N dx_i\,\exp\left(-\frac{1}{2}\lambda_{ij}x_ix_j\right)f(x+a)}{\int_{\mathbf{R}^N}\prod_{i=1}^N dx_i\,\exp\left(-\frac{1}{2}\lambda_{ij}x_ix_j\right)}, \tag{3.2}$$

for positive definite $N\times N$ real matrix λ, then one can obtain the formally equivalent expression to eq. (3.1)

$$\exp\left(i\tilde{S}_M(V,\varphi_i,\bar{\varphi}_i)\right) = \left\{\int\prod_i\mathcal{D}[\varphi'_i]\mathcal{D}[\bar{\varphi}'_i]\exp i\Big[S_{N=4}(V,\varphi_i+\varphi'_i,\bar{\varphi}_i+\bar{\varphi}'_i;g_0)+\right. \tag{3.3}$$

$$\left.+\frac{\tilde{M}}{2}\int\varphi'^2_i+\text{h.c.}\Big]\right\}\left\{\int\prod_i\mathcal{D}[\varphi'_i]\mathcal{D}[\bar{\varphi}'_i]\exp i\Big[\frac{\tilde{M}}{2}\int\varphi'^2_i+\text{h.c.}\Big]\right\}^{-1},$$

where the "reduced" mass $\tilde{M}$ is defined by $\tilde{M}^{-1}\doteq M_0^{-1}-M^{-1}$. The normalization factor in eq. (3.3) is important. In particular, in the limit $M\to M_0$, the numerator above should diverge as $|\delta M|\doteq|M-M_0|\to 0$.

Indeed, by rescaling anomaly

$$\lim_{M\to M_0}\int\prod_i\mathcal{D}[\varphi'_i]\mathcal{D}[\bar{\varphi}'_i]\exp i\left\{S_{N=4}(V,\varphi_i+\varphi'_i,\bar{\varphi}_i+\bar{\varphi}'_i;g_0)+\frac{\tilde{M}}{2}\int\varphi'^2_i+\text{h.c.}\right\}\sim \\ \sim\exp i\left\{S_{N=4}(V,\varphi_i,\bar{\varphi}_i;g_0)+\frac{1}{16}\int\frac{3t_2(A)}{8\pi^2}W^2\log\left(\frac{|\delta M|}{M_0}\right)+\text{h.c.}\right\}. \tag{3.4}$$

3.2 Zinn Justin's transformation

To obtain eq. (3.3) in more rigorous fashion, one applies the transformation introduced in [12]. Note that in what follows the gaussian integral of the type

$$f(\mu;J_i)\doteq\int\prod_{i=1}^3\mathcal{D}[\varphi_i]\,\mathcal{D}[\bar{\varphi}_i]\exp i\left\{\frac{\mu}{2}\int\varphi_i^2+\text{h.c.}+\int J_i\varphi_i+\text{h.c.}\right\} \tag{3.5}$$

inherently depends on the external gauge superfield V (or W). One can evaluate such an "anomalous" gaussian integral by interpreting $f(\mu; J_i)$ as

$$f(\mu; J_i) = \lim_{\epsilon\to 0} f_\epsilon(\mu; J_i) \tag{3.6}$$
$$f_\epsilon(\mu; J_i) \doteq \int \prod_{i=1}^{3} \mathcal{D}[\varphi_i]\,\mathcal{D}[\bar\varphi_i] \exp i \left\{ \epsilon \int d^4\theta\, \bar\varphi_i e^V \varphi_i + \frac{\mu}{2}\int \varphi_i^2 + \text{h.c.} + \int J_i\varphi_i + \text{h.c.} \right\}.$$

From this definition, one can show that

$$\frac{f(\mu; J_i)}{f(\mu; 0)} = \exp\left(\frac{-i}{2\mu}\int J_i^2 + \text{h.c.}\right), \tag{3.7}$$

$$\frac{f(\mu_2; 0)}{f(\mu_1; 0)} = \exp\left\{\frac{-i}{16}\int \frac{3t_2(A)}{8\pi^2} W_a^\alpha W_\alpha^a \log\left(\frac{\mu_2}{\mu_1}\right) + \text{h.c.}\right\}. \tag{3.8}$$

Eq. (3.8) can be trivially obtained if one applies the rescaling transformations $\varphi_i = \sqrt{\frac{\mu_0}{\mu_a}}\varphi_i'$, $a = 1, 2$ and makes use of Konishi anomaly (2.11).

More convincingly, one can calculate the matrix element of the mass operator following the method, e.g., of [13] (see appendix A). Now let us consider the particular example of eq. (3.7)

$$\begin{aligned} &\int \prod_{i=1}^{3} \mathcal{D}[\varphi_i']\,\mathcal{D}[\bar\varphi_i'] \exp i \left[\frac{1}{2}\int \left(M + \tilde M\right)\varphi_i'^2 + \text{h.c.} + \int \varphi_i' \left(-\tilde M\varphi_i + \frac{M}{M_0}J_i\right) + \text{h.c.}\right] \times \\ &\quad\times \left\{\int \prod_{i=1}^{3} \mathcal{D}[\varphi_i']\,\mathcal{D}[\bar\varphi_i'] \exp i \left[\frac{1}{2}\int \left(M + \tilde M\right)\varphi_i'^2 + \text{h.c.}\right]\right\}^{-1} = \\ &= \exp i \left[\frac{1}{2}\int \left(M_0 - \tilde M\right)\varphi_i^2 + \text{h.c.} + \int J_i\varphi_i + \text{h.c.} + \frac{1}{2}\int \left(\frac{1}{M_0} - \frac{M}{M_0^2}\right) J_i^2 + \text{h.c.}\right]. \end{aligned} \tag{3.9}$$

With the help of eq. (3.9) some of the factors defining Z_0 can be written as

$$\begin{aligned} &\exp i \left[\frac{1}{2M_0}\int J_i^2 + \text{h.c.}\right] \exp i \left[\frac{M_0}{2}\varphi_i^2 + \text{h.c.} + \int J_i\varphi_i + \text{h.c.}\right] = \\ &\quad = \exp i \left[\frac{M}{2M_0^2}\int J_i^2 + \frac{\tilde M}{2}\int \varphi_i^2 + \text{h.c.}\right] \left\{\int \prod_{i=1}^{3} \mathcal{D}[\varphi_i']\,\mathcal{D}[\bar\varphi_i'] \times \right. \\ &\qquad \left. \times \exp i \left[\frac{1}{2}\int \left(M + \tilde M\right)\varphi_i'^2 + \text{h.c.} + \int \varphi_i' \left(-\tilde M\varphi_i + \frac{M}{M_0}J_i\right) + \text{h.c.}\right]\right\} \times \\ &\qquad \times \left\{\int \prod_{i=1}^{3} \mathcal{D}[\varphi_i']\,\mathcal{D}[\bar\varphi_i'] \exp i \left[\frac{1}{2}\int \left(M + \tilde M\right)\varphi_i'^2 + \text{h.c.}\right]\right\}^{-1}. \end{aligned} \tag{3.10}$$

Substituting eq. (3.10) into eq. (2.22) and eq. (2.5), one obtains

$$\tilde{Z}_0(J_i,\bar{J}_i,J_V) = \exp i\left[\frac{M}{2M_0^2}\int J_i^2 + \text{h.c.}\right]\int \mathcal{D}[V]\left\{\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i]\,\mathcal{D}[\bar{\varphi}_i]\times\right.$$
$$\times\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i']\,\mathcal{D}[\bar{\varphi}_i']\exp i\left[S_{N=4}(V,\varphi_i,\bar{\varphi}_i;g_0)+\frac{M}{2}\int\varphi_i'^2+\text{h.c.}+\right.$$
$$\left.+\frac{\tilde{M}}{2}\int(\varphi_i-\varphi_i')^2+\text{h.c.}+\frac{M}{M_0}\int J_i\varphi_i'+\text{h.c.}+\int J_V V\right]\times$$
$$\left.\times\left\{\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i']\,\mathcal{D}[\bar{\varphi}_i']\exp i\left[\frac{1}{2}\int\left(M+\tilde{M}\right)\varphi_i'^2+\text{h.c.}\right]\right\}^{-1}\right\}. \quad (3.11)$$

Defining new variables $\varphi_i'' \doteq \varphi_i - \varphi_i'$ and assuming $\mathcal{D}[\varphi_i]\,\mathcal{D}[\bar{\varphi}_i] = \mathcal{D}[\varphi_i'']\,\mathcal{D}[\bar{\varphi}_i'']$ the RHS becomes

$$\exp i\left[\frac{M}{2M_0^2}\int J_i^2+\text{h.c.}\right]\int\mathcal{D}[V]\left\{\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i']\,\mathcal{D}[\bar{\varphi}_i']\exp i\left[\frac{M}{2}\int\varphi_i'^2+\text{h.c.}+\right.\right.$$
$$\left.+\frac{M}{M_0}\int J_i\varphi_i'+\text{h.c.}+\int J_V V\right]\times\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i'']\,\mathcal{D}[\bar{\varphi}_i'']\exp i\left[S_{N=4}(V,\varphi_i'+\varphi_i'',\bar{\varphi}_i'+\bar{\varphi}_i'';g_0)+\right.$$
$$\left.\left.+\frac{\tilde{M}}{2}\int\varphi_i''^2+\text{h.c.}\right]\left\{\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i']\,\mathcal{D}[\bar{\varphi}_i']\exp i\left[\frac{1}{2}\int\left(M+\tilde{M}\right)\varphi_i'^2+\text{h.c.}\right]\right\}^{-1}\right\}. \quad (3.12)$$

This is nothing but the definition of $\tilde{Z}_M$ (see eq. (2.7) and eq. (2.22)) if one identifies

$$\exp\left(iS_M(V,\varphi_i,\bar{\varphi}_i)\right) \equiv \int\prod_{i=1}^{3}\mathcal{D}[\varphi_i']\,\mathcal{D}[\bar{\varphi}_i']\times \quad (3.13)$$
$$\times\exp i\left[S_{N=4}(V,\varphi_i+\varphi_i',\bar{\varphi}_i+\bar{\varphi}_i';g_0)+\frac{\tilde{M}}{2}\int\varphi_i'^2+\text{h.c.}\right]\times$$
$$\times\left\{\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i']\,\mathcal{D}[\bar{\varphi}_i']\exp i\left[\frac{1}{2}\int\left(M+\tilde{M}\right)\varphi_i'^2+\text{h.c.}\right]\right\}^{-1}.$$

eq. (3.13) differs from eq. (3.3) only by the normalization factor. We rewrite, therefore, the RHS as

$$\frac{\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i']\,\mathcal{D}[\bar{\varphi}_i']\exp i\left[\frac{\tilde{M}}{2}\int\varphi_i'^2+\text{h.c.}\right]}{\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i']\,\mathcal{D}[\bar{\varphi}_i']\exp i\left[\frac{1}{2}\int\left(M+\tilde{M}\right)\varphi_i'^2+\text{h.c.}\right]}\times \quad (3.14)$$
$$\times\frac{\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i']\,\mathcal{D}[\bar{\varphi}_i']\exp i\left[S_{N=4}(V,\varphi_i+\varphi_i',\bar{\varphi}_i+\bar{\varphi}_i';g_0)+\frac{\tilde{M}}{2}\int\varphi_i'^2+\text{h.c.}\right]}{\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i']\,\mathcal{D}[\bar{\varphi}_i']\exp i\left[\frac{\tilde{M}}{2}\int\varphi_i'^2+\text{h.c.}\right]}.$$

The first factor is equal, according to eq. (3.8), to

$$\exp\left[\frac{i}{16}\int\frac{3t_2(A)}{8\pi^2}W_a^\alpha W_\alpha^a\log\left(\frac{M+\tilde{M}}{\tilde{M}}\right)+\text{h.c.}\right]. \tag{3.15}$$

But

$$\frac{M+\tilde{M}}{\tilde{M}}=1+\frac{M}{\tilde{M}}=1+M\left(\frac{1}{M_0}-\frac{1}{M}\right)=\frac{M}{M_0}. \tag{3.16}$$

Thus eq. (3.14) is equivalent to

$$\begin{aligned}\exp\left(iS_M(V,\varphi_i,\bar{\varphi}_i)\right)=\exp&\left[\frac{i}{16}\int\frac{3t_2(A)}{8\pi^2}W_a^\alpha W_\alpha^a\log\left(\frac{M}{M_0}\right)+\text{h.c.}\right]\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i']\,\mathcal{D}[\bar{\varphi}_i']\times\\ &\times\exp i\left[S_{N=4}(V,\varphi_i+\varphi_i',\bar{\varphi}_i+\bar{\varphi}_i';g_0)+\frac{\tilde{M}}{2}\int\varphi_i'^2+\text{h.c.}\right]\times\\ &\times\left\{\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i']\,\mathcal{D}[\bar{\varphi}_i']\exp i\left[\frac{\tilde{M}}{2}\int\varphi_i'^2+\text{h.c.}\right]\right\}^{-1}.\end{aligned} \tag{3.17}$$

We have recovered eq. (2.25) with $\tilde{S}_M$ given indeed by eq. (3.3)!

Note that eq. (2.25) or, equivalently, eq. (3.17) is exact and neither depends on perturbation expansion nor it receives "order $\frac{1}{M}$" correction as the authors of [1, 2] appear to hint.

Eq. (3.17) also shows the relationship of Wilsonian action S_M to generating functional of connected part $W(J_i,\bar{J}_i,J_V;\tilde{M})$. Such a relationship is well known in the ERG method with conventional cut-off (see [12]).

4. Conclusions

Our main result is eq. (2.25), which shows clearly the "anomaly origin" of Shifman-Veinstein result. We agree completely with the conclusions of [1, 2], but give a more precise definition of the "anomalous" and "normal" part of the Wilsonian action. While in the above refs the predominance of the "anomalous" part is correctly emphasized, we put the residual (normal) part on the exact footing, writing the equation (Polchinski's) obeyed by it.

Note that the remark about the relevance of "vacuum energy" is not limited to the particular (and peculiar) regularization scheme which we have adopted. Had one applied the usual cut-off method [3] with or without some prescriptions to guarantee the gauge invariance, one would have ended up with similar anomalous terms through the vector field rescaling anomaly (really the rescaling anomaly of accompanying F-P ghosts. See ref. [2]). This also raises the question about the non supersymmetric theory such as the gauge coupling of left handed fermions, where the chiral anomaly [14] would appear like a rescaling anomaly.

These problems can be investigated with more confidence when the problem of incorporating gauge invariance in ERG is solved.

Recently, T. Morris has proposed a novel analysis of the gauge problem in EGR method [6]. His prescription is still to be fully worked out and at present rather cumbersome for the applications. However, the insights which this method appears to confer make us hope that we are getting near the right solution at last.

It would be very interesting to study the significance of "vacuum energy term" in this new approach.

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A. The evaluation of the gaussian integral

Let

$$\frac{\int \prod_{i=1}^{3} \mathcal{D}[\varphi_i]\,\mathcal{D}[\bar{\varphi}_i] \exp i \left[\frac{\mu_2}{2}\int d^2\theta\, \varphi_i^2 + \text{h.c.}\right]}{\int \prod_{i=1}^{3} \mathcal{D}[\varphi_i]\,\mathcal{D}[\bar{\varphi}_i] \exp i \left[\frac{\mu_1}{2}\int d^2\theta\, \varphi_i^2 + \text{h.c.}\right]} = \frac{f(\mu_2)}{f(\mu_1)}\,. \tag{A.1}$$

As we suggested in section 3, this should be interpreted as the limit

$$\lim_{\epsilon\to 0} \frac{\int \prod_{i=1}^{3} \mathcal{D}[\varphi_i]\,\mathcal{D}[\bar{\varphi}_i] \exp i \left[\epsilon \int d^4\theta\, \bar{\varphi}_i e^V \varphi_i + \frac{\mu_2}{2}\int d^2\theta\, \varphi_i^2 + \text{h.c.}\right]}{\int \prod_{i=1}^{3} \mathcal{D}[\varphi_i]\,\mathcal{D}[\bar{\varphi}_i] \exp i \left[\epsilon \int d^4\theta\, \bar{\varphi}_i e^V \varphi_i + \frac{\mu_1}{2}\int d^2\theta\, \varphi_i^2 + \text{h.c.}\right]} = \lim_{\epsilon\to 0} \frac{f_\epsilon(\mu_2)}{f_\epsilon(\mu_1)}\,. \tag{A.2}$$

Let us consider instead

$$\begin{aligned}
\left\langle \frac{\mu}{2}\int d^2\theta\, \varphi_i^2 + \text{h.c.} \right\rangle_\epsilon \doteq{}& \int \prod_{i=1}^{3} \mathcal{D}[\varphi_i]\,\mathcal{D}[\bar{\varphi}_i] \left(\frac{\mu}{2}\int d^2\theta\, \varphi_i^2 + \text{h.c.}\right) \times \\
& \times \exp i \left[\epsilon \int d^4\theta\, \bar{\varphi}_i e^V \varphi_i + \frac{\mu}{2}\int d^2\theta\, \varphi_i^2 + \text{h.c.}\right] \times \\
& \times \Bigg\{ \int \prod_{i=1}^{3} \mathcal{D}[\varphi_i]\,\mathcal{D}[\bar{\varphi}_i] \times \\
& \quad \times \exp i \left[\epsilon \int d^4\theta\, \bar{\varphi}_i e^V \varphi_i + \frac{\mu}{2}\int d^2\theta\, \varphi_i^2 + \text{h.c.}\right] \Bigg\}^{-1} = \\
={}& -i\mu \frac{\partial}{\partial \mu} f_\epsilon(\mu)\, [f_\epsilon(\mu)]^{-1}\,.
\end{aligned} \tag{A.3}$$

The matrix element can be explicitly evaluated with Feynman graphs (see [13]). The result is

$$\left\langle \frac{\mu}{2}\int d^2\theta\,\varphi_i^2 + \text{h.c.}\right\rangle_\epsilon = -\frac{1}{16}\int \frac{3t_2(A)}{8\pi^2}W^2 + \text{h.c.} + \mathcal{O}\left(\left(\frac{\epsilon}{\mu}\right)^4\right). \tag{A.4}$$

Integrating the differential equation for $f_\epsilon(\mu)$

$$\frac{f_\epsilon(\mu_2)}{f_\epsilon(\mu_1)} = \exp\left[-\frac{i}{16}\int \frac{3t_2(A)}{8\pi^2}W^2\log\left(\frac{\mu_2}{\mu_1}\right) + \text{h.c.} + \mathcal{O}\left(\left(\frac{\epsilon}{\mu}\right)^4\right)\right]. \tag{A.5}$$

Taking the limit $\epsilon \to 0$

$$\frac{f(\mu_2)}{f(\mu_1)} = \exp\left[-\frac{i}{16}\int \frac{3t_2(A)}{8\pi^2}W^2\log\left(\frac{\mu_2}{\mu_1}\right) + \text{h.c.}\right]. \tag{A.6}$$

This is the result we wanted to prove.

B. φ^2 insertion

In this appendix,[2] we consider the effect of introducing the gauge invariant source term

$$\int d^4x\, d^2\theta\, K(x,\theta)\sum_{i=1}^{3}\varphi_i^2(x,\theta) \tag{B.1}$$

instead of $\int J_i\varphi_i$. We consider the regularized $N = 1$ SUSY Yang-Mills partition function (cf. eq. 2.5)

$$\begin{aligned} Z_0'(J_i,\bar{J}_i,K,\bar{K},H,\bar{H},J_V) = \int \mathcal{D}[V]\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i]\,\mathcal{D}[\bar{\varphi}_i]\exp i\Big[&S_{N=4}(V,\varphi,\bar{\varphi};g_0) + \\ &+\frac{M_0}{2}\int\varphi_i^2 + \int J_i\varphi_i + \text{h.c.} + \int J_V V + \\ &+\frac{1}{2}\int K\varphi_i^2 + \text{h.c.} + \int H\frac{3}{16}\frac{t_2(A)}{8\pi^2}W^2 + \text{h.c.}\Big]. \end{aligned} \tag{B.2}$$

With respect to the original definition eq. (2.5), we have added the source terms for the composite (gauge invariant) operators φ_i^2, W^2 and their conjugates.

J_i, $\bar{J}_i$, K, $\bar{K}$, H, $\bar{H}$ and J_V are space-time dependent superfields.

Varying the regularizing mass M_0 to $M < M_0$, we would like to see if its effect can be expressed by the simple modification of eq. (2.7)

$$\begin{aligned} Z_M'(\tilde{J}_i,\bar{\tilde{J}}_i,\tilde{K},\bar{\tilde{K}},\tilde{H},\bar{\tilde{H}},J_V) = \int \mathcal{D}[V]\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i]\,\mathcal{D}[\bar{\varphi}_i]\exp i\Big[&\tilde{S}_M(V,\varphi,\bar{\varphi}) + \\ &+\frac{M}{2}\int\varphi_i^2 + \text{h.c.} + \int\tilde{J}_i\,\varphi_i + \text{h.c.} + \int J_V V + \\ &+\frac{1}{2}\int\tilde{K}\varphi_i^2 + \text{h.c.} + \int\tilde{H}\frac{3}{16}\frac{t_2(A)}{8\pi^2}W^2 + \text{h.c.}\Big]. \end{aligned} \tag{B.3}$$

[2]This calculation has been suggested by G.C. Rossi.

In eq. (B.3) one assumes that the "Wilsonian action" $\tilde{S}_M$ does not depend on the source terms $\tilde{J}_i$, $\bar{\tilde{J}}_i$, $\tilde{K}$, $\bar{\tilde{K}}$, $\tilde{H}$, $\bar{\tilde{H}}$ and J_V.

The new source superfields $\tilde{J}_i$, $\tilde{K}$, $\tilde{H}$ and their adjoints depend on the mass parameter M and the original J_i, K and H. They satisfy the initial conditions

$$\begin{aligned} \tilde{J}_i(M = M_0) &= J_i \\ \tilde{K}(M = M_0) &= K \\ \tilde{H}(M = M_0) &= H \,. \end{aligned} \tag{B.4}$$

For the path integral of (B.3) the "trivial" equality eq. (2.8) is still valid if one writes

$$S_{tot} = \tilde{S}_M + \frac{M}{2}\int \varphi_i^2 + \int \tilde{J}_i\,\varphi_i + \text{h.c.} + \int J_V V + \frac{1}{2}\int \tilde{K}\varphi_i^2 + \int \tilde{H}\frac{3}{16}\frac{t_2(A)}{8\pi^2}W^2 + \text{h.c.} \tag{B.5}$$

As in section 2, we separate first the mass term as well as φ_i, $\bar{\varphi}_i$ and V source terms. Then eq. (2.8) takes the form

$$\begin{aligned} 0 = \int d^4p\,\frac{1}{M}\int \mathcal{D}[V]\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i]\,\mathcal{D}[\bar{\varphi}_i]\Big\{\int d^2\theta\,\Big[-\frac{i}{2}\delta_{ii} + \\ +\frac{1}{2}\left(i\frac{\delta^2 S_M^{'}}{\delta\varphi_i(p)\delta\varphi_i(-p)} - \frac{\delta S_M^{'}}{\delta\varphi_i(p)}\frac{\delta S_M^{'}}{\delta\varphi_i(-p)}\right) + \frac{M^2}{2}\varphi_i(-p)\varphi_i(p) + \\ +\frac{1}{2}\tilde{J}_i(-p)\tilde{J}_i(p) + M\tilde{J}_i(-p)\,\varphi_i(p)\Big] + \text{h.c.}\Big\}\exp\left(iS_{tot}\right), \end{aligned} \tag{B.6}$$

with $S_M^{'} \doteq \tilde{S}_M + \frac{1}{2}\int \tilde{K}\varphi_i^2 + \text{h.c.} + \int \tilde{H}\frac{3}{16}\frac{t_2(A)}{8\pi^2}W^2 + \text{h.c.}$

Further separating $\tilde{S}_M$ from $S_M^{'}$, it becomes

$$\begin{aligned} \int \mathcal{D}[V]\int\prod_{i=1}^{3}\mathcal{D}[\varphi_i]\,\mathcal{D}[\bar{\varphi}_i]\left\{\frac{3}{16}\int\frac{t_2(A)}{8\pi^2}W^2\left(\frac{M+\tilde{K}}{M}\right) + \right. \\ +\frac{1}{2M}\int\left(M+\tilde{K}\right)^2\varphi_i^2 + \frac{1}{2M}\int\left(i\frac{\delta^2\tilde{S}_M}{\delta\varphi_i(p)\delta\varphi_i(-p)} - \frac{\delta\tilde{S}_M}{\delta\varphi_i(p)}\frac{\delta\tilde{S}_M}{\delta\varphi_i(-p)}\right) + \\ \left. +\frac{1}{2M}\int\tilde{J}_i(-p)\tilde{J}_i(p) + \text{h.c.}\right\}\exp\left(iS_{tot}\right) = 0\,, \end{aligned} \tag{B.7}$$

where we have replaced the "anomaly" $i\sum_i \delta_{ii}$ by Konishi estimate $-\frac{3}{8}\int\frac{t_2(A)}{8\pi^2}W^2$ (cf. eq. (2.14)).

Note that the anomaly is now contributed also by the double derivative of the composite source term $\frac{\delta^2}{\delta\varphi_i^2(x)}\frac{1}{2}\int\tilde{K}\varphi_i^2(x)$.

From eq. (B.7) one obtains the new conditions for M-invariance

$$M\partial_M\tilde{S}_M = \frac{1}{2M}\int\left(-\frac{\delta\tilde{S}_M}{\delta\varphi_a^i(p)}\frac{\delta\tilde{S}_M}{\delta\varphi_a^i(-p)} + i\frac{\delta^2\tilde{S}_M}{\delta\varphi_a^i(p)\delta\varphi_a^i(-p)}\right); \tag{B.8}$$

$$M\partial_M\tilde{H} = \frac{\tilde{K}+M}{M} \qquad \text{with} \qquad \tilde{H}(M=M_0) = H\,; \tag{B.9}$$

$$M\partial_M \frac{1}{2}\left(\tilde{K}+M\right) = \frac{1}{2M}\left(\tilde{K}+M\right)^2 \quad \text{with} \quad \tilde{K}(M=M_0)=K\,; \tag{B.10}$$

$$\left.\begin{array}{r}\tilde{J}_i = \tilde{f}(M) J_i\,, \\ M\partial_M \tilde{f} = \frac{1}{M}\left(\tilde{K}+M\right)\tilde{f}\end{array}\right\} \quad \text{with} \quad \tilde{f}(M=M_0)=1\,. \tag{B.11}$$

We see that the renormalized sources $\tilde{H}$ and $\tilde{J}_i$ depend on $\tilde{K}$, the source of φ_i^2 term.

From the condition (B.10)

$$M\partial_M \frac{1}{\tilde{K}+M} = M\partial_M \frac{1}{M} \tag{B.12}$$

i.e.

$$\frac{1}{\tilde{K}+M} - \frac{1}{M} = const = \frac{1}{K+M_0} - \frac{1}{M_0}\,. \tag{B.13}$$

Thus $\tilde{K}(x,\theta;M)$ depends on M and the initial source factor $K(x,\theta)$ in "non multiplicative" way

$$\frac{1}{\tilde{K}(x,\theta;M)+M} = \frac{1}{K(x,\theta)+M_0} - \frac{1}{\tilde{M}}\,. \tag{B.14}$$

From the condition (B.11)

$$\partial_M \log \tilde{f} = \frac{1}{M^2}\left(\frac{1}{K+M_0} + \frac{1}{M} - \frac{1}{M_0}\right)^{-1} \tag{B.15}$$

i.e.

$$\tilde{f}(M) = \frac{M}{M_0}\left[1 + \frac{M}{M_0}\left(\frac{1}{M} - \frac{1}{M_0}\right)K\right]^{-1}\,. \tag{B.16}$$

The condition (B.9) is equivalent to the equation for $\log \tilde{f}(M)$ above

$$\tilde{H}(x,\theta;M) = H(x,\theta) + \log\frac{M}{M_0} - \log\left[1 + \frac{M}{M_0}\left(\frac{1}{M} - \frac{1}{M_0}\right)K\right]. \tag{B.17}$$

If the conditions (B.8), (B.14), (B.15) and (B.17) are satisfied, eq. (B.7) implies that the invariance equation analogous to eq. (2.21) holds.

$$\begin{aligned} M\partial_M &\left\{\exp i\left[\frac{M}{2M_0}\int \frac{1}{K(x)+M_0}\left(1 - \frac{M}{M_0}\frac{K(x)}{\tilde{M}}\right)^{-1} J_i^2(x) + h.c.\right] Z'_M\right\} \doteq \\ &\doteq M\partial_M \tilde{Z}'_M = 0. \end{aligned} \tag{B.18}$$

Again Z'_M is unchanged except for the calculable tree level contribution. The extra factor disappears if one considers the case $J_i = \bar{J}_i = 0$.
On the other hand, the results (B.14), (B.15) and (B.17) imply the mixing of operators, in particular φ_i^2 and W^2, when the regularizing mass M is varied.

From eqs (B.14), (B.15) and (B.17)

$$\begin{aligned}
\frac{\partial \tilde K}{\partial K} &= \left(\frac{M}{M_0}\right)^2 \left(1-\frac{M}{M_0}\frac{K}{\tilde M}\right)^{-2}\\
\frac{\partial \tilde H}{\partial K} &= \frac{M}{M_0}\frac{1}{\tilde M}\left(1-\frac{M}{M_0}\frac{K}{\tilde M}\right)^{-1}\\
\frac{\partial \tilde F}{\partial K} &= \left(\frac{M}{M_0}\right)^2 \frac{1}{\tilde M}\left(1-\frac{M}{M_0}\frac{K}{\tilde M}\right)^{-2}.
\end{aligned} \tag{B.19}$$

If one takes the configurations with $J_i = \bar J_i = 0$, then

$$\begin{aligned}
\left\langle \frac{1}{2}\sum_{i=1}^{3}\varphi_i^2\right\rangle_{M_0} &= \frac{1}{iZ_0}\frac{\delta Z_0}{\delta K} = \frac{1}{iZ_M}\left(\frac{\partial \tilde K}{\partial K}\frac{\delta}{\delta \tilde K} + \frac{\partial \tilde H}{\partial K}\frac{\delta}{\delta \tilde H}\right) Z_M =\\
&= \left(\frac{M}{M_0}\right)^2\left(1-\frac{M}{M_0}\frac{K}{\tilde M}\right)^{-2}\left\langle \frac{1}{2}\sum_{i=1}^{3}\varphi_i^2\right\rangle_M +\\
&\quad + \frac{M}{M_0}\frac{1}{\tilde M}\left(1-\frac{M}{M_0}\frac{K}{\tilde M}\right)^{-1}\left\langle \frac{3}{16}\frac{t_2(A)}{8\pi^2}W^2\right\rangle_M
\end{aligned} \tag{B.20}$$

i.e.

$$\begin{aligned}
\left\langle \frac{M_0}{2}\sum_{i=1}^{3}\varphi_i^2\right\rangle_{M_0} &= \frac{M}{M_0}\left(1-\frac{M}{M_0}\frac{K}{\tilde M}\right)^{-2}\left\langle \frac{M}{2}\sum_{i=1}^{3}\varphi_i^2\right\rangle_M + \left(1-\frac{M}{M_0}\right)\times\\
&\quad\times\left(1-\frac{M}{M_0}\frac{K}{\tilde M}\right)^{-1}\left\langle -\frac{3}{16}\frac{t_2(A)}{8\pi^2}W^2\right\rangle_M .
\end{aligned} \tag{B.21}$$

On the other hand, from eq. (B.17)

$$\left\langle -\frac{3}{16}\frac{t_2(A)}{8\pi^2}W^2\right\rangle_{M_0} = \left\langle -\frac{3}{16}\frac{t_2(A)}{8\pi^2}W^2\right\rangle_M . \tag{B.22}$$

For the "physical" configurations, i.e. for $J_i = \bar J_i = K = \bar K = H = \bar H = 0$ and arbitrary J_V

$$\left\langle \frac{M_0}{2}\sum_{i=1}^{3}\varphi_i^2\right\rangle_{M_0} = \frac{M}{M_0}\left\langle \frac{M}{2}\sum_{i=1}^{3}\varphi_i^2\right\rangle_M + \left(1-\frac{M}{M_0}\right)\left\langle -\frac{1}{16}\frac{3t_2(A)}{8\pi^2}W^2\right\rangle_M , \tag{B.23}$$

or, conversely,

$$\left\langle \frac{M}{2}\sum_{i=1}^{3}\varphi_i^2\right\rangle_{M} = \frac{M_0}{M}\left\langle \frac{M_0}{2}\sum_{i=1}^{3}\varphi_i^2\right\rangle_{M_0} + \left(1-\frac{M_0}{M}\right)\left\langle -\frac{1}{16}\frac{3t_2(A)}{8\pi^2}W^2\right\rangle_{M_0} . \tag{B.24}$$

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